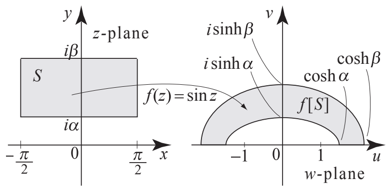
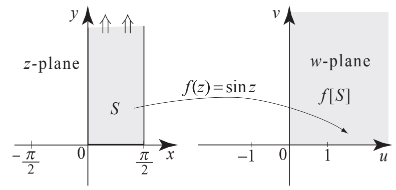
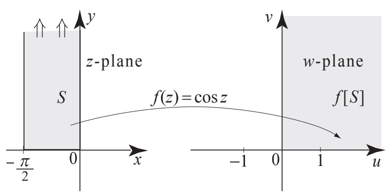
17.



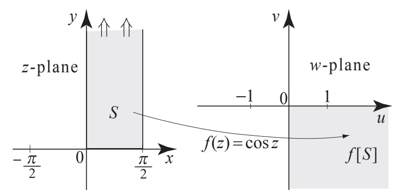
18.



19.



20.



21. Establish identities (1.7.17) and (1.7.19).

22. Let S be the horizontal strip $\{z = x + iy;\ x \ge 0\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$. Find the image of S by the mapping $f(z) = \sinh z$. [Hint: Express $\sinh z$ in terms of $\sin z$.]

23. We study properties of the mapping $z \mapsto \sin z$.
(a) Show that the half-line $x = \frac{\pi}{2}$, $y \ge 0$, is mapped to the half-line $u \ge 1$, $v = 0$.
(b) Show that the half-line $x = \frac{-\pi}{2}$, $y \ge 0$, is mapped to the half-line $u \le -1$, $v = 0$.
(c) Conclude that the boundary of the set S in Example 1.7.8 is mapped to the boundary of $f[S]$.
(d) Recall from your calculus course that an ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $0 < b < a$ has its foci at $x = \pm\sqrt{a^2 - b^2}$. Show that all the ellipses in Example 1.7.8 have the same foci located on the u-axis at $u = \pm 1$.

24. (Zeros of hyperbolic functions) Let $z \in \mathbb{C}$. Show that

$$\sinh z = 0 \quad \Leftrightarrow \quad z = ik\pi,\ k \text{ an integer;}$$

and

$$\cosh z = 0 \quad \Leftrightarrow \quad z = i\Big(k + \frac{1}{2}\Big)\pi,\ k \text{ an integer.}$$

[Hint: z is a zero of $\sin z \Leftrightarrow iz$ is a zero of the hyperbolic sine (why?). Reason in the same way for the cosine.]

25. (Linearization) Let m and n be nonnegative integers such that $m + n = p$. We discuss how to express the product $\cos^m\theta \sin^n\theta$, as a linear combination of terms involving $\cos(j\theta)$ and $\sin(k\theta)$, where $1 \le j,\ k \le p$. For example, the identity $\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$ is called the linearization of $\cos^3\theta$. Derive this identity by raising both sides of (1.7.3) to the third power and simplifying.

26. Linearize $\sin^4\theta$.

In Exercises 27–52*, establish the stated identities. Here z_1, z_2, z are complex numbers and $z = x + iy$ with x, y real. Working with hyperbolic functions, you may want to use the corresponding one for trigonometric functions and* (1.7.30) *and* (1.7.31).

27. (a) $\sin(-z) = -\sin z$ (b) $\sin(z + 2\pi) = \sin z$

28. (a) $\cos(z + \pi) = -\cos z$ (b) $\sin(z + \pi) = -\sin z$