mapping f are streamlines $\Psi(w) = C$. Since the real axis is a streamline for ψ and Γ is the image of the real axis, we conclude that Γ is a streamline for Ψ. Thus Ψ is a stream function for Ω.

In the following examples, we take the simple stream function $\psi(z) = y$ for the upper half-plane. Streamlines for the region Ω are found by using Proposition 7.4.8.

Example 7.4.11. (Fluid flow in a sector) Find and plot the streamlines for the sector in Figure 7.88, where fluid flows in along the line $\operatorname{Arg} w = \frac{\pi}{4}$ and flows out along $\operatorname{Arg} w = 0$.

Solution. From Example 7.4.3, the Schwarz-Christoffel transformation $f(z) = z^{\frac{1}{4}}$ maps the upper half-plane to the sector $0 < \arg z < \frac{\pi}{4}$. We use the simple stream function in the upper half-plane $\psi(z) = y$ to generate a solution. Streamlines in the z-plane are parametrized as $\gamma(x) = x + iy_0$ for fixed y_0. Streamlines in the w-plane are images of these under f; we have $f(\gamma(x)) = (x + iy_0)^{\frac{1}{4}}$. As the parameter x increases, the streamlines are traced in the manner shown in Figure 7.88. Fluid comes in along $\operatorname{Arg} w = \frac{\pi}{4}$ and out along $\operatorname{Arg} w = 0$. □

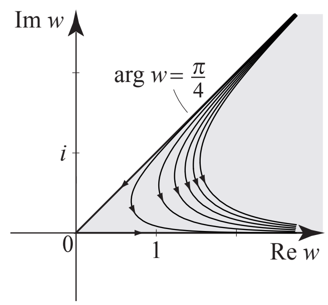

Fig. 7.88 Streamlines in a sector.

Example 7.4.12. (Fluid flow in the doubly slit plane) Find and plot the streamlines for the doubly slit plane in Figure 7.83, where fluid flows in from the upper left, past the double obstacle, and flows out to the lower left.

Solution. In Example 7.4.7, we found that $f(z) = -\frac{1}{\pi}z^2 + \frac{2}{\pi}\operatorname{Log} z + \frac{1}{\pi} - i$ is a conformal mapping of the upper half-plane onto the doubly slit plane Ω. Taking $\psi(z) = y$ to be the stream function for the upper half-plane, for each $y_0 \geq 0$ we have a streamline parametrized by

$$\gamma(x) = x + iy_0, \quad -\infty < x < \infty.$$

By Proposition 7.4.8, the curves $f(\gamma(x))$, $-\infty < x < \infty$, are streamlines in the doubly slit plane. They are

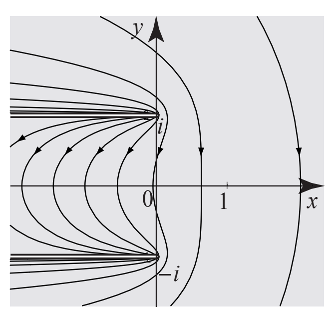

Fig. 7.89 Streamlines in a doubly slit plane.