

- 7.** Let C be a positively oriented closed simple path, let z_0 be a complex number in the interior of C , and let $p(z) = \sum_{j=1}^{n-1} a_j z^j$ be a polynomial of degree $n-1$ for some integer $n \geq 1$. Show that

$$\int_C \frac{p(z-z_0)}{(z-z_0)^n} dz = 2\pi i a_{n-1}.$$

- 8. Evaluate**

$$\int_C \left[\frac{i}{z-1} + \frac{6}{(z-1)^2} \right] dz$$

in the following cases: (a) 1 is inside C ; (b) 1 is outside C .

In Exercises 9–12 evaluate the path integral over the curve γ parametrized by $\gamma(t) = 1 + i + 2e^{it}$, $0 \leq t \leq 2\pi$.

9. $\int_\gamma \frac{dz}{z-1}$

10. $\int_\gamma \frac{dz}{(z-i)(z-1)}$

11. $\int_\gamma \frac{dz}{(z-3i)(z-1)}$

12. $\oint_\gamma \frac{dz}{z^2+9}$

In Exercises 12–32, evaluate the integrals. Indicate clearly how you are applying previously established results.

12. $\int_{[z_1, z_2, z_3, z_1]} \sin(z^2) dz$, where $z_1 = 0, z_2 = -i, z_3 = 1$.

13. $\int_{C_1(0)} \frac{e^z}{z+2} dz$.

14. $\int_\gamma (z^2 + 2z + 3) dz$, where γ is an arbitrary path joining 0 to 1.

15. $\int_{C_1(i)} \left(\frac{z-1}{z+1} \right)^2 z dz$.

16. $\int_\gamma \frac{3i}{z-2i} dz$, where $\gamma(t) = e^{it} + \frac{e^{2it}}{2}$, $0 \leq t \leq 2\pi$.

17. $\int_\gamma \frac{e^z}{z+i} dz$, where $\gamma(t) = i + e^{it}$, $0 \leq t \leq 2\pi$.

18. $\int_{C_1(0)} \frac{1}{z-\frac{1}{2}} dz$.

19. $\int_{C_1(0)} \frac{1}{(z-\frac{1}{2})^2} dz$.

20. $\int_{C_4(0)} \left((z-2+i)^2 + \frac{i}{z-2+i} - \frac{3}{(z-2+i)^2} \right) dz$.

21. $\int_{[z_1, z_2, z_3, z_1]} z^2 \operatorname{Log} z dz$, where $z_1 = 1, z_2 = 1+i, z_3 = -1+i$.

22. $\int_{C_1(i)} \frac{1}{(z-i)(z+i)} dz$.

23. $\int_{C_3(i)} \frac{1}{(z-i)(z+i)} dz$.

24. $\int_{C_{\frac{3}{2}}(1+i)} \frac{1}{(z-1)(z-i)(z+i)} dz$.

25. $\int_{C_2(0)} \frac{z}{z^2-1} dz$.

26. $\int_{C_2(0)} \frac{1}{z^2+1} dz$.

27. $\int_{C_{\frac{3}{2}}(0)} \frac{z^2+1}{(z-2)(z+1)} dz$.