**Solution.** In order to apply Theorem 3.3.4, we must find an antiderivative of $\frac{1}{z}$ that is analytic in a region that contains the path $[z_1, z_2, z_3]$. We cannot use $\operatorname{Log} z$ as antiderivative, because it is not analytic in a region that contains the path $[z_1, z_2, z_3]$. Instead, we will use a different branch of the logarithm. We know from Example 3.3.2(d) that $\log_\alpha z$ is an antiderivative of $\frac{1}{z}$ in the region $\Omega_\alpha$ ($\mathbb{C}$ minus the ray at angle $\alpha$). Choose $\alpha$ in such a way that the branch cut of $\log_\alpha z$ does not intersect the path $[z_1, z_2, z_3]$.

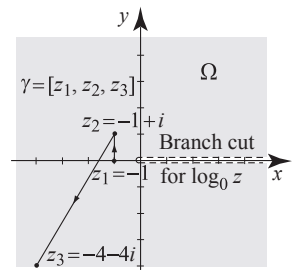


**Fig. 3.32** Picture for Example 3.3.7

Taking, for example, $\alpha = 0$ we write

$$\log_0 z = \ln|z| + i \arg_0 z,$$

where $0 < \arg_0 z \le 2\pi$. By Theorem 3.3.4 we have

$$\begin{aligned}\int_{[z_1,z_2,z_3]} \frac{1}{z}\, dz &= \log_0(z_3) - \log_0(z_1)\\ &= \ln|-4-4i| + i\arg_0(-4-4i) - (\ln 1 + i\arg_0(-1))\\ &= \frac{1}{2}\ln 32 + i\frac{5\pi}{4} - i\pi = \frac{5}{2}\ln 2 + i\frac{\pi}{4}.\end{aligned}$$

Thus the value of the integral is equal to $\frac{5}{2}\ln 2 + i\frac{\pi}{4}$. □

## *Integrals over Closed Paths*

We now turn to applications of Theorem 3.3.4 related to integrals over closed paths. We start with some straightforward ones.

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**Example 3.3.8. (Integrals over closed paths)**
(a) Since $z^2/2$ is an antiderivative of $z$ on the plane, if $\gamma$ is a closed path, then by Theorem 3.3.4,

$$\int_\gamma z\, dz = 0.$$

(b) Likewise,

$$\int_\gamma e^{2iz}\, dz = 0,$$

because $e^{2iz}$ has an antiderivative $\frac{e^{2iz}}{2i}$ for all $z$ in the plane.