The ~~graph of~~ curve $\gamma$ is simply the graph of $f$ over the interval $[-\pi, \pi]$. For $t \neq 0$, we have

$$f'(t) = 2t\sin\frac{1}{t} - \cos\frac{1}{t},$$

and for $t = 0$

$$f'(0) = \lim_{t\to 0}\frac{t^2\sin\frac{1}{t}}{t} = 0$$

by the squeeze theorem. So $f$ is continuous and differentiable on the closed intervals $[-\pi, 0]$ and $[0, \pi]$ (with one-sided limits at the endpoints). But $f'$ is not continuous on $[-\pi, 0]$ or on $[0, \pi]$; thus $\gamma$ is not a path. The graphs of $f$ and $f'$ are shown in Figure 3.14. $\square$

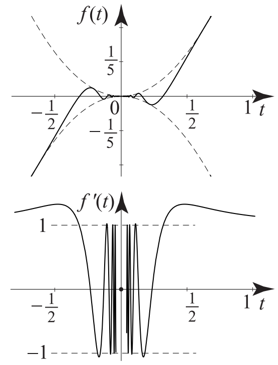


**Fig. 3.14** $f$ is a differentiable function but its derivative is not continuous at a point. Thus the graph of $f$ is not a piecewise continuous curve, i.e., a path.

# Exercises 3.1

*In Exercises* 1–8 *parametrize the curves over suitable intervals* $[a, b]$.

**1.** The line segment with initial point $z_1 = 1 + i$ and terminal point $z_2 = -1 - 2i$.

**2.** The line segment through the origin as initial point and terminal point $z = e^{i\frac{\pi}{3}}$.

**3.** The counterclockwise circle with center at $3i$ and radius 1.

**4.** The clockwise circle with center at $-2 - i$ and radius 3.

**5.** The positively oriented arc on the unit circle such that $-\frac{\pi}{4} \leq \operatorname{Arg} z \leq \frac{\pi}{4}$.

**6.** The negatively oriented arc on the unit circle such that $-\frac{\pi}{4} \leq \operatorname{Arg} z \leq \frac{\pi}{4}$.

**7.** The closed polygonal path $[z_1, z_2, z_3, z_1]$ where $z_1 = 0$, $z_2 = i$, and $z_3 = -1$.

**8.** The polygonal path $[z_1, z_2, z_3, z_4]$ where $z_1 = 1$, $z_2 = 2$, $z_3 = i$, and $z_4 = 2i$.

*In Exercises* 9–11*, describe the parametrizations of the paths shown in the figures.*

**9.**

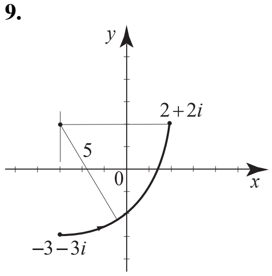


**Fig. 3.15** Circular arc of radius 5 centered at $-3+2i$.

**10.**

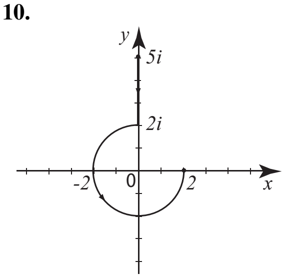


**Fig. 3.16** Vertical line segment followed by circular arc of radius 2 centered at 0.

**11.**

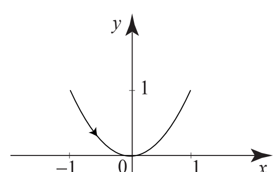


**Fig. 3.17** Arc of the parabola $y = x^2$ for $-1 \leq x \leq 1$.

**12.** Find a parametrization for the reverse of the path in Exercise 9.