Definition 3.1.10. A **path** or a **contour** is a curve γ defined on a closed interval $[a, b]$ which is continuously differentiable or piecewise continuously differentiable. The path γ is **closed** if $\gamma(a) = \gamma(b)$.

Definition 3.1.11. Given points $a_0 < a_1 < \cdots < a_m$ and paths γ_j on $[a_{j-1}, a_j]$, $j = 1, \dots, m$, such that $\gamma_j(a_j) = \gamma_{j+1}(a_j)$ for all $j = 1, \dots, m-1$, the combined path

$$\Gamma = [\gamma_1, \dots, \gamma_m]$$

is piecewise defined on $[a_0, a_m]$ by $\Gamma(t) = \gamma_j(t)$ for $t \in [a_{j-1}, a_j]$, $j = 1, \dots, m$.

Thus, according to Definitions 3.1.10 and 3.1.11, a path or a contour γ is a finite sequence of continuously differentiable curves, $\gamma_1, \gamma_2, \dots, \gamma_m$, joined at the endpoints, i.e., $\gamma = [\gamma_1, \dots, \gamma_m]$. The path γ is closed if the initial point of γ_1 coincides the terminal point of γ_m, i.e., $\gamma_1(a_0) = \gamma_m(a_m)$.

Example 3.1.12. The path $\gamma = [\gamma_1, \gamma_2, \gamma_3]$ in Figure 3.10 consists of the curves: The line segment $\gamma_1 = [-2, -1]$; the semicircle γ_2; and the line segment $\gamma_3 = [1, 2]$. We can parametrize γ by the interval $[-2, 2]$ as follows:

$$\gamma(t) = \begin{cases} t & \text{if} \quad -2 \le t \le -1, \\ e^{i\frac{\pi}{2}(1-t)} & \text{if} \quad -1 \le t \le 1, \\ t & \text{if} \quad 1 \le t \le 2. \end{cases}$$

The choice of the interval $[-2,2]$ as the domain of definition was just for convenience. Other closed intervals can be used to parametrize γ. □

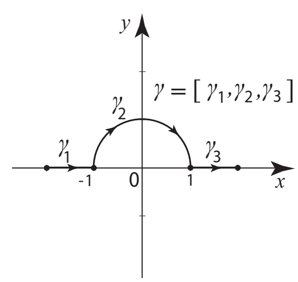

Fig. 3.10 The path of Example 3.1.12.

Example 3.1.13. (Polygonal paths) A **polygonal path**, $\gamma = [z_1, z_2, \dots, z_n]$ is the union of the line segments $[z_1, z_2]$, $[z_2, z_3]$, $\dots$, $[z_{n-1}, z_n]$. This is a piecewise linear path with initial point z_1 and terminal point z_n and may have self intersections.

A polygonal path is called **simple** if it does not have self intersections, except possibly at the endpoints, that is, z_1 and z_n may coincide. The polygonal path is called **closed** if $z_1 = z_n$. As an illustration, let $z_1 = 0$, $z_2 = 1 + i$, and $z_3 = -1 + i$; then $\gamma = [z_1, z_2, z_3, z_1]$ is a simple closed polygonal path. To find the equation of γ, we start by finding the equations of the paths γ_1, γ_2, and γ_3, shown in Figure 3.11. From Example 3.1.2(d), we have

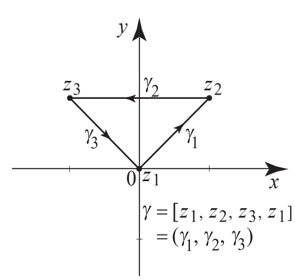

Fig. 3.11 The closed polygonal path $[z_1, z_2, z_3, z_1]$