Also by the definitions of E_ε and $U(f;\Phi)^{\varepsilon,N}$, for any $x \in E_\varepsilon$ we have

$$t\big|\nabla(\Phi_t * f)(\xi)\big|\Big(\frac{t}{t+\varepsilon}\Big)^N \frac{1}{(1+\varepsilon|\xi|)^N} \le K M_1^*(f;\Phi)^{\varepsilon,N}(x) \tag{2.1.33}$$

for all ξ satisfying $|\xi - x| < t < \frac{1}{\varepsilon}$. It follows from (2.1.32) and (2.1.33) that

$$t\big|\nabla(\Phi_t * f)(\xi)\big| \le 2K\big|(\Phi_t * f)(y_0)\big|\left(\frac{1+\varepsilon|\xi|}{1+\varepsilon|y_0|}\right)^N \tag{2.1.34}$$

for all ξ satisfying $|\xi - x| < t < \frac{1}{\varepsilon}$. We let $z \in B(x,t)\cap B(y_0,t)$. Applying the mean value theorem and using (2.1.34), we obtain, for some ξ between y_0 and z,

$$\begin{aligned}
\big|(\Phi_t * f)(z) - (\Phi_t * f)(y_0)\big| &= \big|\nabla(\Phi_t * f)(\xi)\big|\,|z-y_0| \\
&\le \frac{2K}{t}\big|(\Phi_t * f)(\xi)\big|\left(\frac{1+\varepsilon|\xi|}{1+\varepsilon|y_0|}\right)^N |z-y_0| \\
&\le \frac{2^{N+1}K}{t}\big|(\Phi_t * f)(y_0)\big|\,|z-y_0| \\
&\le \frac{1}{2}\big|(\Phi_t * f)(y_0)\big|,
\end{aligned}$$

provided z also satisfies $|z-y_0| < 2^{-N-2}K^{-1}t$ in addition to $|z-x| < t$. Therefore, for z satisfying $|z-y_0| < 2^{-N-2}K^{-1}t$ and $|z-x| < t$ we have

$$\big|(\Phi_t * f)(z)\big| \ge \frac{1}{2}\big|(\Phi_t * f)(y_0)\big| \ge \frac{1}{4}M_1^*(f;\Phi)^{\varepsilon,N}(x),$$

where the last inequality uses (2.1.32). Thus we have

$$\begin{aligned}
M\big(M(f;\Phi)^q\big)(x) &\ge \frac{1}{|B(x,t)|}\int_{B(x,t)} \big[M(f;\Phi)(w)\big]^q\,dw \\
&\ge \frac{1}{|B(x,t)|}\int_{B(x,t)\cap B(y_0,2^{-N-2}K^{-1}t)} \big[M(f;\Phi)(w)\big]^q\,dw \\
&\ge \frac{1}{|B(x,t)|}\int_{B(x,t)\cap B(y_0,2^{-N-2}K^{-1}t)} \frac{1}{4^q}\big[M_1^*(f;\Phi)^{\varepsilon,N}(x)\big]^q\,dw \\
&\ge \frac{|B(x,t)\cap B(y_0,2^{-N-2}K^{-1}t)|}{|B(x,t)|}\frac{1}{4^q}\big[M_1^*(f;\Phi)^{\varepsilon,N}(x)\big]^q \\
&\ge C'(n,N,K)^{-1}4^{-q}\big[M_1^*(f;\Phi)^{\varepsilon,N}(x)\big]^q,
\end{aligned}$$

where we used the simple geometric fact that if $|x-y_0| \le t$ and $\delta > 0$, then

$$\frac{|B(x,t)\cap B(y_0,\delta t)|}{|B(x,t)|} \ge c_{n,\delta} > 0,$$

the minimum of this constant being obtained when $|x-y_0| = t$. See Figure 2.1.