$1 < p_1, p_2 < 2$ and $2 < p = (1/p_1 + 1/p_2)^{-1} < \infty$; this was shown by Grafakos and Li [161]. The analogous result fails in dimensions $n \geq 2$ when p_1, p_2, or p' is greater than 2, as shown by Diestel and Grafakos [120]. The last reference contains a version of de Leeuw's theorem in the multilinear setting (Exercise 7.3.10).

The m-linear version of the Calderón–Zygmund theorem (Theorem 7.4.6) is due to Grafakos and Torres [177], although the bilinear case was independently obtained by Kenig and Stein [219]. Both sets of authors employed the Calderón–Zygmund decomposition, which first appeared in the trilinear setting as a lemma in the work of Coifman and Meyer [91]. Proposition 7.4.8 (the m-linear Peetre–Spanne–Stein theorem) and a version of Theorem 7.4.9 is also from [177]. The latter provides a $T(1)$ type theorem for m-linear operators associated with Calderón–Zygmund kernels. Other results of this type are due to Christ and Journé [84], Bényi, Demeter, Nahmod, Thiele, Torres, and Villaroya [23], Bényi [22], and Hart [186]. The action of multilinear Calderón–Zygmund singular integrals on Hardy spaces was studied by Grafakos and Kalton [159].

The case $m = 1$ of Theorem 7.5.5 is essentially contained in Hörmander's article [193]. The m-linear version was obtained by Tomita [351] when the target index is L^p for $p > 1$ and $r = 2$ (Corollary 7.5.9). The extension to indices $p \leq 1$ is due to Grafakos and Si [170], while the extension to the endpoint cases where L^∞ is allowed in the domain (but not in all spaces) is due to Grafakos, Miyachi, and Tomita [168]. Miyachi and Tomita [267] extended Theorem 7.5.5 to situations where Lebesgue spaces are replaced by Hardy spaces in the domain and obtained minimal smoothness conditions for the multipliers; see also the related work [268]. The formulation of Theorem 7.5.5 in the text was suggested by Tomita and is natural according to the viewpoint of a corresponding theorem in Kurtz and Wheeden [232]. Theorem 7.5.3 was first proved by Coifman and Meyer [92] via Fourier series techniques; see Coifman and Meyer [93] for extensions. Bernicot and Germain [30] obtained boundedness for bilinear multipliers whose symbols have narrow support.

Gilbert and Nahmod [154] obtained boundedness for bilinear multipliers on $\mathbf{R} \times \mathbf{R}$ whose derivatives of order α blow up like the distance to a line (with slope not taking three values) raised to the power $-|\alpha|$. Muscalu, Tao, and Thiele [280] provided an analogous m-linear version of this result. A maximal function related to the bilinear Hilbert transform was shown by Lacey [234] to be bounded on the same products of spaces as the bilinear Hilbert transform. More general singular multilinear maximal operators were studied by Demeter, Tao, and Thiele [118]. Deep counterexamples for trilinear operators were devised by Christ [82] and Demeter [117]. On the topic of multilinear Littlewood–Paley theory, one may consult the articles of Lacey [233], Diestel [119], Bernicot [29], Bernicot and Shrivastava [32], and Mohanty and Shrivastava [270], [271].

Paraproducts provide important examples of multilinear operators with specific properties. They first emerged in Bony's theory of paradifferential operators [41], which took the pseudodifferential operator theory of Coifman and Meyer [93] a step further. The boundedness of paraproducts on L^p spaces for $p > 1$ is easily achieved via duality, but the extension to indices $p \leq 1$ is more delicate and was proved independently by Grafakos and Kalton [158] and by Auscher, Hofmann, Muscalu, Thiele, and Tao [10]; subsequently this result was reproved by Lacey and Metcalfe [235], while a different proof was given by Bényi, Maldonado, Nahmod, and Torres [24]. The articles of Bernicot [28], Bilyk, Lacey, Li, Wick [38] Muscalu, Pipher, Thiele, and Tao [278], [279] study certain forms of paraproducts in depth. The expository article of Bényi, Maldonado, Naibo [25] makes a strong case for the use of paraproducts in analysis and partial differential equations.

A large body of literature on the topic of multilinear weighted norm inequalities appeared after the initial work of Grafakos and Torres [175]. A natural class of multiple weights that satisfies a vector A_p condition suitable for the multilinear Calderón–Zygmund theory was developed by Lerner, Ombrosi, Pérez, Torres and Trujillo-González [241]. Other weighted estimates were obtained by Bui and Duong [51], Hu [198], Li, Xue, and Yabuta [243]. Fujita and Tomita [147] and Li and Sun [242] obtained weighted estimates for multilinear Fourier multipliers.

The commutator estimate $\|\mathcal{J}_s(fg) - f\mathcal{J}_s(g)\|_{L^p} \leq C\|\nabla f\|_{L^\infty}\|\mathcal{J}_{s-1}(g)\|_{L^p} + C\|\mathcal{J}_s(f)\|_{L^p}\|g\|_{L^\infty}$, where $1 < p < \infty$ and $s > 0$, was proved by Kato and Ponce [210], where $\mathcal{J}_s = (1-\Delta)^{s/2}$ is the Bessel potential on $\mathbf{R}^n$. Kenig, Ponce, Vega [218] obtained the homogeneous commutator estimate $\|D^s(fg) - fD^s f - gD^s f\|_{L^r} \leq C\|D^{s_1} f\|_{L^p}\|D^{s_2} g\|_{L^q}$, with $D^s = (-\Delta)^{s/2}$ in place of $\mathcal{J}_s$, with $s = s_1 + s_2$ for $s, s_1, s_2 \in (0,1)$, and $1 < p, q, r < \infty$ satisfying $1/r = 1/p + 1/q$. The inequality in