Given a distribution $f \in \dot{F}_p^{\alpha,q}$, using identity (2.3.13), we write

$$f = \sum_{j\in\mathbf{Z}} \Psi_{2^{-j}} * \Theta_{2^{-j}} * f\,,$$

where the convergence is in $\mathscr{S}'(\mathbf{R}^n)/\mathscr{P}(\mathbf{R}^n)$ in view of Corollary 1.1.7.

For each Q in $\mathscr{D}_j$ define a constant

$$s_Q = |Q|^{\frac{1}{2}} \sup_{y\in Q} |(\Psi_{\ell(Q)} * f)(y)| \sup_{|\gamma|\le L+1} \big\|\partial^\gamma \Theta\big\|_{L^1}$$

and a function

$$a_Q(x) = \frac{1}{s_Q} \int_Q \Theta_{\ell(Q)}(x-y)(\Psi_{\ell(Q)} * f)(y)\,dy\,. \tag{2.3.14}$$

It is straightforward to verify that a_Q is supported in $3Q$ and that it has vanishing moments up to and including order L, since Θ does so. Moreover, using (2.3.14) we obtain for all $|\gamma| \le L+1$

$$|\partial^\gamma a_Q| \le \frac{1}{s_Q} \big\|\partial^\gamma \Theta\big\|_{L^1} \ell(Q)^{-|\gamma|} \sup_Q |\Psi_{\ell(Q)} * f| \le |Q|^{-\frac{1}{2}-\frac{|\gamma|}{n}}\,,$$

which makes the function a_Q a smooth L-atom.

Using this notation, we write

$$f = \sum_{j\in\mathbf{Z}} \sum_{Q\in\mathscr{D}_j} \int_Q \Theta_{2^{-j}}(x-y)(\Psi_{2^{-j}} * f)(y)\,dy = \sum_{j\in\mathbf{Z}} \Big(\sum_{Q\in\mathscr{D}_j} s_Q a_Q \Big)\,,$$

where the series in j converges in $\mathscr{S}'(\mathbf{R}^n)/\mathscr{P}(\mathbf{R}^n)$.

Let b be as in (2.3.9). Now note that

$$\begin{aligned}
&\sum_{\ell(Q)=2^{-j}} \big(|Q|^{-\frac{\alpha}{n}-\frac{1}{2}} s_Q \chi_Q(x)\big)^q \\
&\qquad = C \sum_{\ell(Q)=2^{-j}} \big(2^{j\alpha} \sup_{y\in Q} |(\Psi_{2^{-j}} * f)(y)| \chi_Q(x)\big)^q \\
&\qquad \le C \sup_{|z|\le\sqrt{n}2^{-j}} \big(2^{j\alpha}(1+2^j|z|)^{-b} |(\Psi_{2^{-j}} * f)(x-z)|\big)^q (1+2^j|z|)^{bq} \\
&\qquad \le C\big(2^{j\alpha} M^{**}_{b,j}(f,\Psi)(x)\big)^q\,,
\end{aligned}$$

where we used the fact that in the first inequality there is only one nonzero term in the sum because of the appearance of the characteristic function. Summing over all $j \in \mathbf{Z}^n$, raising to the power $1/q$, and taking L^p norms yields the estimate

$$\big\|\{s_Q\}_Q\big\|_{\dot{f}_p^{\alpha,q}} \le C \Big\| \Big(\sum_{j\in\mathbf{Z}} |2^{j\alpha} M^{**}_{b,j}(f;\Psi)|^q \Big)^{\frac{1}{q}} \Big\|_{L^p} \le C \big\|f\big\|_{\dot{F}_p^{\alpha,q}}\,,$$