$$
\begin{aligned}
v \ge \delta t^\gamma \implies (f_t)^*(v) &= \inf\big\{s>0:\ d_{f_t}(s) \le v\big\} \\
&\le \inf\big\{s>0:\ d_f(s)\le v\big\} \qquad\qquad \text{since } d_{f_t} \le d_f \\
&= f^*(v),
\end{aligned}
$$

$$
\begin{aligned}
v < \delta t^\gamma \implies (f_t)^*(v) &= \inf\big\{s>0:\ d_{f_t}(s) \le v\big\} \\
&\le f^*(\delta t^\gamma), \qquad \text{since } f^*(\delta t^\gamma) \in \big\{s>0:\ d_{f_t}(s)\le v\big\}.
\end{aligned}
$$

We summarize these observations in a couple of inequalities:

$$
\begin{aligned}
(f^t)^*(s) \ &\le \begin{cases} f^*(s) & \text{if } 0<s<\delta t^\gamma, \\ 0 & \text{if } s \ge \delta t^\gamma, \end{cases} \\
(f_t)^*(s) \ &\le \begin{cases} f^*(\delta t^\gamma) & \text{if } 0<s<\delta t^\gamma, \\ f^*(s) & \text{if } s \ge \delta t^\gamma. \end{cases}
\end{aligned}
$$

It follows from these inequalities that f^t lies in $L^{p_0,m}$ and f_t lies in $L^{p_1,m}$ for all $t>0$.

The quasi-linearity of the operator T and (1.4.9) imply

$$
\begin{aligned}
&\big\|T(f)\big\|_{L^{q,r}} \\
&= \big\|t^{\frac{1}{q}} T(f)^*(t)\big\|_{L^r(\frac{dt}{t})} \\
&\le K\big\|t^{\frac{1}{q}} (|T(f_t)|+|T(f^t)|)^*(t)\big\|_{L^r(\frac{dt}{t})} \\
&\le K\big\|t^{\frac{1}{q}} T(f_t)^*(\tfrac{t}{2}) + t^{\frac{1}{q}} T(f^t)^*(\tfrac{t}{2})\big\|_{L^r(\frac{dt}{t})} \\
&\le K a_r\Big(\big\|t^{\frac{1}{q}} T(f_t)^*(\tfrac{t}{2})\big\|_{L^r(\frac{dt}{t})} + \big\|t^{\frac{1}{q}} T(f^t)^*(\tfrac{t}{2})\big\|_{L^r(\frac{dt}{t})}\Big) \\
&\le K \max\{1, 2^{\frac{1}{r}-1}\}\Big(\big\|t^{\frac{1}{q}} T(f_t)^*(\tfrac{t}{2})\big\|_{L^r(\frac{dt}{t})} + \big\|t^{\frac{1}{q}} T(f^t)^*(\tfrac{t}{2})\big\|_{L^r(\frac{dt}{t})}\Big). \qquad (1.4.31)
\end{aligned}
$$

It follows from (1.4.30) that

$$
t^{\frac{1}{q_0}} T(f^t)^*(\tfrac{t}{2}) \ \le 2^{\frac{1}{q_0}} \sup_{s>0} s^{\frac{1}{q_0}} T(f^t)^*(s) \le 2^{\frac{1}{q_0}} M_0' \big\|f^t\big\|_{L^{p_0,m}}, \qquad (1.4.32)
$$

$$
t^{\frac{1}{q_1}} T(f_t)^*(\tfrac{t}{2}) \ \le 2^{\frac{1}{q_1}} \sup_{s>0} s^{\frac{1}{q_1}} T(f_t)^*(s) \le 2^{\frac{1}{q_1}} M_1' \big\|f_t\big\|_{L^{p_1,m}}, \qquad (1.4.33)
$$

for all $t>0$. Now use (1.4.32), (1.4.33), and the facts that

$$
t^{\frac{1}{q}} T(f^t)^*(\tfrac{t}{2}) = t^{\frac{1}{q}-\frac{1}{q_0}} t^{\frac{1}{q_0}} T(f^t)^*(\tfrac{t}{2}) \le t^{\frac{1}{q}-\frac{1}{q_0}} 2^{\frac{1}{q_0}} M_0' \big\|f^t\big\|_{L^{p_0,m}}
$$

$$
t^{\frac{1}{q}} T(f_t)^*(\tfrac{t}{2}) = t^{\frac{1}{q}-\frac{1}{q_1}} t^{\frac{1}{q_1}} T(f_t)^*(\tfrac{t}{2}) \le t^{\frac{1}{q}-\frac{1}{q_1}} 2^{\frac{1}{q_1}} M_1' \big\|f_t\big\|_{L^{p_1,m}},
$$