for some constant C, and since for a given $x \in \Omega$, $\Phi(x)$ is the sum of at most 12^n functions with nonzero values, it follows that

$$\left|\partial_1^{q_1} \cdots \partial_n^{q_n} \Phi(x)\right| \le C_n \, \ell_k^{-(q_1+\cdots+q_n)}$$

when $x \in Q_k^*$ and thus

$$\left|\frac{\partial^\beta}{\partial x^\beta} \frac{1}{\Phi(x)}\right| \le C'_{n,\beta} \, \ell_k^{-|\beta|}$$

for $x \in Q_k^*$. We conclude that for every multiindex α there is a constant $C_{\alpha,n}$ such that

$$\left|\frac{\partial^\alpha}{\partial x^\alpha} \varphi_k(x)\right| \le C_{\alpha,n} \, \ell_k^{-|\alpha|} \qquad x \in Q_k^* .$$

More on Whitney decompositions can be found in the article of Whitney [373] and the books of Stein [338], Krantz and Parks [204].