for $k \geq 1$. Note that for $k \geq 1$ we have $\nu(Y)^{\frac{1}{k+1}} \leq \max(1, \nu(Y))^{\frac{1}{2}}$ and consider the cases $\mu(S_k) \geq 3^{-k-1}$ and $\mu(S_k) \leq 3^{-k-1}$ when summing in $k \geq 1$. The term with $k = 0$ is easier.]

1.3.8. Prove that for $0 < x < 1$ we have

$$\frac{\sin(\pi x)}{2} \int_{-\infty}^{+\infty} \frac{1}{\cosh(\pi t) + \cos(\pi x)}\, dt = x\,,$$
$$\frac{\sin(\pi x)}{2} \int_{-\infty}^{+\infty} \frac{1}{\cosh(\pi t) - \cos(\pi x)}\, dt = 1 - x\,,$$

and conclude that Lemma 1.3.8 reduces to Lemma 1.3.5 when the functions $M_0(y)$ and $M_1(y)$ are constant and assumption (1.3.29) is replaced by the stronger assumption that F is bounded on $\overline{S}$.
[*Hint:* In the first integral write $\cosh(\pi t) = \frac{1}{2}(e^{\pi t} + e^{-\pi t})$. Then use the change of variables $s = e^{\pi t}$.]

1.3.9. Let (X, μ), (Y, ν) be σ-finite measure spaces, and let $0 < p_0 < p_1 \leq \infty$. Let T be a sublinear operator defined on the space $L^{p_0}(X) + L^{p_1}(X)$ and taking values in the space of measurable functions on Y. Suppose that T ~~is a sublinear operator such that~~ maps L^{p_0} to L^∞ with constant A_0 and L^{p_1} to L^∞ with constant A_1. Prove that T maps L^p to L^∞ with constant $2A_0^{1-\theta} A_1^{\theta}$ where $0 < \theta < 1$ and

$$\frac{1-\theta}{p_0} + \frac{\theta}{p_1} = \frac{1}{p}\,.$$

1.4 Lorentz Spaces

Suppose that f is a measurable function on a measure space (X, μ). It would be desirable to have another function f^* defined on $[0, \infty)$ that is decreasing and *equidistributed* with f. By this we mean

$$d_f(\alpha) = d_{f^*}(\alpha) \tag{1.4.1}$$

for all $\alpha \geq 0$. This is achieved via a simple construction discussed in this section.

1.4.1 Decreasing Rearrangements

Definition 1.4.1. Let f be a complex-valued function defined on X. The *decreasing rearrangement* of f is the function f^* defined on $[0, \infty)$ by

$$f^*(t) = \inf\{s > 0 : \ d_f(s) \leq t\} = \inf\{s \geq 0 : \ d_f(s) \leq t\}\,. \tag{1.4.2}$$